Amplitude Bounds in the Ladder Graph Approximation*

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We consider forward scattering in the ladder approximation for a trilinear scalar interaction. The corresponding Bethe-Salpeter integral equation for the absorptive part of the amplitude is of the Volterra type; and the kernel and inhomogeneous term are both positive. We exploit these special features in order to set upper bounds on the absorptive amplitude for arbitrary values of the coupling constant g. Two different techniques are described. For large scattering energies the bounds obtained imply corresponding bounds on the value of the leading Regge pole $\alpha(0)$. In the limit of weak coupling our upper bound on $\alpha(0)$ is linear in g^2 and in fact coincides exactly with the known weak-coupling result. In the limit of strong coupling our upper bound varies as the square root of g^2. The correctness of this feature is discussed on the analogy with the Schrödinger problem of binding in a potential field.

I. INTRODUCTION

THE study of ladder graphs for scattering processes has for some time been a very popular occupation.[1–11] The summation over an infinite subset of ladder graphs leads to an amplitude which satisfies the simplest integral equation of the Bethe-Salpeter type.[12,13] One hopes that the ladder approximation already reveals many of the features of a complete theory. It has been used for the study of bound-state questions; and viewed from a different channel, it has been used as a model for high-energy scattering ("multiperipheral model").[6–9] Jointly these two aspects make the ladder approximation an interesting testing ground for the Regge hypothesis in conventional field theory.[5,7,10]

A vast amount of work has been done on the mentioned integral equation concerning the existence of solutions and their analytic properties with respect to energy and angular momentum variables. It should also be interesting, however, to have some information on the numerical *size* of the scattering solutions. This is the purpose of the present paper, in which for simplicity we study forward scattering in a trilinear scalar interaction theory. In particular we concentrate on the absorptive part of the scattering amplitude. This satisfies a Volterra-type integral equation and we can utilize the positivity of its kernel to obtain an upper bound on the absorptive part, for all values of the scattering energy and of the coupling constant.

Two somewhat different procedures are described for obtaining bounds (Secs. II and III, respectively). The second of the two has considerable flexibility and promise. The upper bound which is obtained, when studied in the limit of high energies, provides an upper bound on the leading Regge pole in the crossed channel. In the limit of weak coupling, $g^2 \to 0$, the bound which we get is linear in g^2 and in fact coincides with the known, exact expression. In the strong coupling limit, where no exact results are available, our upper bound varies as the square root of g^2. We discuss, however, the Regge problem for potentials and are able to show that this feature must be present there in the strong coupling limit, for a wide class of potentials.

II. A SIMPLE MAJORIZATION IN THE INTEGRAL EQUATION

We are concerned with the ladder approximation for forward scattering of two scalar particles, with respective momenta p and k. The ladder diagrams are shown in Fig. 1. The heavy lines describe particles of mass m; the wavy lines (rungs of the ladder) correspond to particles of mass μ. All of the particles are taken to be scalars. The squared barycentric energy is $s=(p+k)^2$ and the invariant momentum transfer is $t=0$. On the mass shell $p^2=k^2=m^2$.

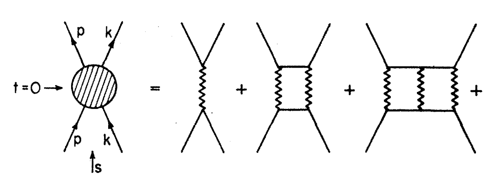

FIG. 1. Ladder diagrams for the forward scattering of two scalar particles.

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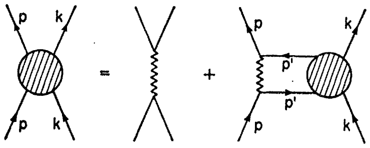

FIG. 2. Diagrams for the absorptive part of the forward amplitude.

The absorptive part of the forward amplitude, as defined for p^2 off the mass shell in the ladder approximation, satisfies the integral equation[6]

$$A(p,k)=\pi g^2\delta[(p+k)^2-\mu^2]+\frac{g^2}{(2\pi)^3}\int d^4p'\frac{A(p',k)}{(m^2-p'^2)^2}\delta[(p-p')^2-\mu^2];\quad(1)$$

this is symbolized by the diagram shown in Fig. 2. In terms of the invariants p^2, s, and $k^2=m^2$, this can be written

$$A(s,p^2)=\pi g^2\delta(s-\mu^2)+\frac{g^2}{16\pi^2}[(s-p^2-m^2)^2-4p^2m^2]^{-1/2}\times\int_{\mu^2}^{[(\sqrt{s})-\mu]^2}ds'\int dp'^2\frac{A(s',p'^2)}{(m^2-p'^2)^2},\quad(2)$$

where the upper and lower limits on the p'^2 integration are given by

$$(p'^2)_\pm=p^2+\mu^2-(1/2s)(s+p^2-m^2)(s-s'+\mu^2)\pm[(s-p^2-m^2)^2-4p^2m^2]^{1/2}\times[(s-s'+\mu^2)^2-4\mu^2s]^{1/2}(1/2s).$$

With $\psi(s,p^2)=A(s,p^2)-\pi g^2\delta(s-\mu^2)$ we then have the equation

$$\psi(s,p^2)=A^{(4)}(s,p^2)+\frac{g^2}{16\pi^2}[(s-p^2-m^2)^2-4p^2m^2]^{-1/2}\int_{4\mu^2}^{[(\sqrt{s})-\mu]^2}ds'\int dp'^2\frac{\psi(s',p'^2)}{(m^2-p'^2)^2},\quad(3)$$

where $A^{(4)}(s,p^2)$ is the fourth-order contribution to the absorptive part coming from the box diagram:

$$A^{(4)}(s,p^2)=(g^4/16\pi)\{s(s-4\mu^2)\}^{1/2}\times\{(s-p^2-m^2)(m^2s-\mu^2p^2-\mu^2m^2)+m^2p^2(s-4\mu^2)+s(\mu^2-m^2)^2\}^{-1}.$$

The integral equation (3) is of the Volterra type and the iteration solution converges for every finite value of the coupling constant g. In fact, for any finite value of s, $\psi(s,p^2)$ can be computed with a finite number of iterations. Moreover, both the inhomogeneous term and the kernel are positive throughout the range of the variables appearing in the integral. It follows that any majorization of the inhomogeneous term and/or of the kernel leads to a new integral equation whose solution is an upper bound for $\psi(s,p^2)$, provided that the iteration series of the majorizing equation still converges pointwise.

It is our purpose to exploit this fact in order to obtain upper bounds on the absorptive part $\psi(s,p^2)$; in particular, thereby, to set upper limits on the asymptotic behavior for $s\to\infty$. In the discussion which follows, we shall assume that $m\leqslant\mu$. This restriction on the masses is for technical convenience and has no other importance. The arguments given could be supplemented to apply, without essential change, to arbitrary ratio of the two masses, provided stability conditions are met.

We shall proceed in two different ways. In this section we majorize the integral equation (3) sufficiently drastically to yield a new equation which is directly soluble. In the next section we discuss a somewhat different, and generally more promising approach, which involves the use of trial functions to achieve majorization.

We now majorize Eq. (3) by employing the inequalities (valid for $p^2\leqslant 0$):

$$A^{(4)}(s,p^2)\leqslant\frac{g^4}{16\pi m^2}\frac{\theta(s-4\mu^2)}{s-p^2-m^2};$$

$$(p'^2)_+\leqslant 0;\quad (p'^2)_-\geqslant-\infty;$$

$$[(s-p^2-m^2)^2-4m^2p^2]^{-1/2}\leqslant(s-p^2-m^2)^{-1}.$$

The majorizing equation for $s\geqslant 4\mu^2$ is then

$$\bar{\psi}(s,p^2)=\frac{g^4}{16\pi m^2}\frac{1}{s-p^2-m^2}+\frac{g^2}{16\pi^2}\frac{1}{s-p^2-m^2}\int_{4\mu^2}^{s}ds'\int_{-\infty}^{0}dp'^2\frac{\bar{\psi}(s',p'^2)}{(m^2-p'^2)^2},\quad(4)$$

where we have also extended the upper limit of the s' integration to the value s.

Introducing

$$f(s)=\int_{-\infty}^{0}dp'^2\frac{\bar{\psi}(s',p'^2)}{(m^2-p'^2)^2},$$

we find

$$f(s)=\left[\frac{1}{m^2(s-2m^2)}+\frac{1}{(s-2m^2)^2}\ln\frac{m^2}{s-m^2}\right]\times\left[\frac{g^4}{16\pi m^2}+\frac{g^2}{16\pi^2}\int_{4\mu^2}^{s}ds'f(s')\right].$$

A further majorization leads to the equation

$$(s-2m^2)f(s)=\frac{g^4}{16\pi m^4}+\frac{g^2}{16\pi^2m^2}\int_{4\mu^2}^{s}ds'f(s'),$$

whose solution is

$$\bar{f}(s)=\frac{g^4}{16\pi m^4}\frac{1}{s-2m^2}\left(\frac{s-2m^2}{4\mu^2-2m^2}\right)^{g^2/16\pi^2m^2}.$$

If we now extend the limits on the p'^2 integration in Eq. (3) from $-\infty$ to 0 and, for simplicity, the s' limits from $4\mu^2$ to s, we may use the above bound on $f(s)$ to deduce an upper bound for ψ on the mass shell, $p^2=m^2$. In this way we find

$$\psi(s,m^2)\leqslant A^{(4)}(s,m^2)+\frac{g^4}{16\pi m^2}[s(s-4m^2)]^{-1/2}\times\left[\left(\frac{s-2m^2}{4\mu^2-2m^2}\right)^{g^2/16\pi^2m^2}-1\right]\theta(s-9\mu^2). \quad (5)$$

Of particular interest are the implications of this bound for large values of s. If the asymptotic behavior of the absorptive amplitude for forward scattering is dominated by a Regge pole $\alpha(t=0)$, our result implies

$$\alpha(0)\leqslant -1+(g^2/16\pi^2m^2). \quad (6)$$

But in the weak coupling limit, $g\rightarrow 0$, expressions have been obtained elsewhere[5,10] for the location of the dominant pole at arbitrary momentum transfer t:

$$\alpha(t)=-1+\frac{g^2}{16\pi^2}\int_0^1\frac{dx}{m^2-x(1-x)t}+(\text{higher order in } g^2).$$

We see therefore that our upper bound, Eq. (6), in fact coincides with the true weak coupling limit for forward scattering. But of course we have shown that (6) is an upper bound for *any* value of the coupling constant. That the bound is such a good one in the weak coupling limit is a source of some surprise, in view of our reckless majorization approximations.

The upper bound, for any value of g, which is represented by Eq. (6) can be related to corresponding questions for the case of a bound state in an attractive Yukawa potential $V=-\lambda e^{-\mu r}/r$.

It is known there that the angular momentum for fixed binding energy B can be expanded in powers of λ according to

$$\alpha(B)=-1+(\lambda/2\sqrt{B})+(\text{higher order in } \lambda)$$

The analog of Eq. (6) would then read

$$\alpha(B)\leqslant -1+(\lambda/2\sqrt{B}) \quad (7)$$

for any λ. However, the correctness of (7) is in fact immediately obvious. The right-hand side of (7) is the known trajectory (as a function of B) for the Coulomb potential $-\lambda/r$. Clearly, a Yukawa potential $-\lambda e^{-\mu r}/r$ is everywhere weaker than the corresponding Coulomb potential; hence for given binding energy it requires a weaker centrifugal barrier. So we expect $\alpha(B)$ for the Yukawa case to be smaller than, or at most equal to, the corresponding $\alpha(B)$ for the Coulomb case.

III. TRIAL FUNCTION METHOD

A more flexible procedure for bounding the absorptive amplitude, and as it turns out a better one in the present problem, can be obtained by a different method which is based on the following simple observation.

Lemma: The inequality

$$\psi(x)-\int_a^b K(x,x')\psi(x')dx'\geqslant 0 \quad [\text{or } \leqslant 0]$$

for x in the interval (a,b) implies $\psi(x)\geq 0$ [or ≤ 0] in (a,b), provided that $K(x,x')\geq 0$ and that the iteration series $(1+K+K^2+\cdots)\psi$ converges pointwise.

We can apply this to obtain upper (or lower) bounds on the solutions of integral equations. For example, if the given equation is

$$\psi(x)=\varphi(x)+\int_a^b K(x,x')\psi(x')dx', \quad K(x,x')\geqslant 0,$$

we try to determine the parameters $\alpha_1, \alpha_2, \cdots$ of a *trial* function $\bar{\psi}(x;\alpha)$ so that—in the case where we seek say an upper bound—

$$\bar{\psi}(x;\alpha)-\int_a^b \bar{K}(x,x')\bar{\psi}(x';\alpha)dx'\geqslant \phi(x) \text{ for all } x \text{ in } (a,b).$$

Here $\bar{K}(x,x')\geq K(x,x')$ may be chosen to simplify integrations, etc.; i.e., we can regard $\bar{K}$ as a "trial" kernel adjusted for convenience provided it bounds the true kernel. For best results of course one chooses $\bar{K}=K$. We now have, symbolically,

$$\bar{\psi}-\bar{K}\bar{\psi}\geq\psi-K\psi$$

or

$$(\bar{\psi}-\psi)-K(\bar{\psi}-\psi)+(K-\bar{K})\bar{\psi}\geq 0,$$

and hence $(\bar{\psi}-\psi)-K(\bar{\psi}-\psi)\geq 0$. Thus, according to the lemma $\bar{\psi}(x;\alpha)$ is an upper bound to $\psi(x)$, provided the iteration series converges pointwise. Clearly analogous procedures apply where one seeks to obtain a lower bound. As long as the kernels are nonsingular the requirement of pointwise convergence is always met for Volterra equations.

In applying the present method to the integral equation (3) we seek a trial function $\bar{\psi}(s,p^2)$ such that

$$\bar{\psi}(s,p^2)-\frac{g^2}{16\pi^2}[(s-p^2-m^2)^2-4m^2p^2]^{-1/2}\times\int_{4\mu^2}^{[(\sqrt{s})-\mu]^2}ds'\int_{(p'^2)_-}^{(p'^2)_+}dp'^2\frac{\bar{\psi}(s',p'^2)}{(m^2-p'^2)^2}\geqslant A^{(4)}(s,p^2). \quad (8)$$

To simplify the subsequent computations, we shall allow ourselves to worsen matters somewhat by further majorizing the kernel (extending the range of integration) and the expression for $A^{(4)}(s,p^2)$. Using the inequalities

$$-s+s'+p^2\leqslant (p'^2)_-\leqslant (p'^2)_+\leqslant p^2\left(\frac{s'-m^2}{s-m^2}\right),$$

valid for $p^2 \leqslant 0$, we write

$$\bar{\psi}(s,p^2) - \frac{g^2}{16\pi^2} \frac{1}{s-p^2-m^2} \times \int_{4\mu^2}^{s} ds' \int_{-s+s'+p^2}^{p^2(s'-m^2)/(s-m^2)} dp'^2 \frac{\bar{\psi}(s',p'^2)}{(m^2-p'^2)^2} \geqslant \frac{g^4}{16\pi^2 m^2} \frac{1}{s-p^2-m^2}. \quad (9)$$

In the present method these kinematic approximations are not necessary, and we would no doubt get better bounds by working directly with (8). But in any case they are less drastic than the approximations which led to Eq. (4), where we were forced to drastic measures in order to obtain a soluble integral equation. Moreover, we conjecture from a detailed consideration of the approximations which lead from (8) to (9) that they are not serious insofar as one is concerned with the asymptotic behavior of the absorptive amplitude in the limit $s \to \infty$.

In terms of the variables

$$u = s - p^2 - m^2,$$
$$r = -p^2/(s-p^2-m^2),$$

we rewrite (9) in the further majorized form

$$\bar{\psi}(u,r) - \frac{g^2}{16\pi^2} \frac{1}{u} \int_{4\mu^2-m^2}^{u} u' du' \int_{r}^{1} dr' \frac{\bar{\psi}(u',r')}{(u'r'+m^2)^2} \geqslant \frac{g^4}{16\pi m^2} \frac{1}{u}. \quad (10)$$

We now adopt as trial function the expression

$$\bar{\psi} = c u^{\alpha} (ur+m^2)^{-\beta}, \quad (11)$$

where c, α, β are parameters to be adjusted. We then require, from (10),

$$c u^{\alpha+1} (ur+m^2)^{-\beta} - \frac{g^2 c}{16\pi^2} \int_{4\mu^2-m^2}^{u} du' u'^{\alpha} \frac{1}{\beta+1} \times \{(u'r+m^2)^{-\beta-1} - (u'+m^2)^{-\beta-1}\} \geqslant \frac{g^4}{16\pi m^2}. \quad (12)$$

In particular, for $u = 4\mu^2 - m^2$ we have the condition

$$c(4\mu^2-m^2)^{\alpha+1} (4\mu^2)^{-\beta} \geq g^4/16\pi m^2.$$

Supposing this to be met, it will now be sufficient to require in addition that the derivative of the left-hand side of (12) with respect to u shall be positive for all r in the interval (0,1) and for $u > 4\mu^2 - m^2$. Dropping, as we may do by way of further majorization, the second term in the curly brackets of Eq. (12), we thus require

$$\frac{\partial}{\partial u}[u^{\alpha+1}(ur+m^2)^{-\beta}] - \frac{g^2}{16\pi^2} \frac{1}{\beta+1} u^{\alpha} (ur+m^2)^{-\beta-1} \geqslant 0,$$

or

$$(\alpha+1-\beta) ur + (\alpha+1) m^2 - \frac{g^2}{16\pi^2} \frac{1}{\beta+1} \geqslant 0.$$

This latter relation is equivalent to the pair of inequalities

$$\begin{aligned} \alpha+1-\beta &\geq 0, \\ \alpha+1 &\geq (g^2/16\pi^2 m^2)[1/(\beta+1)]. \end{aligned} \quad (13)$$

The smallest value of α which is compatible with these inequalities is

$$\alpha_0 = -\tfrac{3}{2} + [\tfrac{1}{4} + (g^2/16\pi^2 m^2)]^{1/2}, \quad (14)$$

with $\beta_0 = \alpha_0 + 1$.

The corresponding upper bound on the absorptive amplitude, for $p^2 \leq 0$, is therefore

$$\psi(s,p^2) \leqslant \frac{g^4}{16\pi m^2} \left(\frac{4\mu^2}{4\mu^2-m^2}\right)^{\alpha_0+1} \frac{(s-p^2-m^2)^{\alpha_0}}{(m^2-p^2)^{\alpha_0+1}}. \quad (15)$$

Finally, in order to obtain an upper bound for the absorptive amplitude $\psi(s,m^2)$ on the mass shell, we substitute the right-hand side of (15) for ψ under the integral in Eq. (3), since there p'^2 runs over negative values only. After some obvious further majorizations we obtain

$$\begin{aligned} \psi(s,m^2) \leqslant{}& A^{(4)}(s,m^2) + \frac{g^2}{16\pi^2} \frac{g^4}{16\pi m^2} \left(\frac{4\mu^2}{4\mu^2-m^2}\right)^{\alpha_0+1} [s(s-4m^2)]^{-1/2} \times \int_{4\mu^2-m^2}^{s-2m^2} du' u'^{\alpha_0} \int_{-\infty}^{0} \frac{dp'^2}{(m^2-p'^2)^{\alpha_0+3}} \\ ={}& A^{(4)}(s,m^2) + \frac{g^4}{16\pi m^2} \left(\frac{4\mu^2}{m^2}\right)^{\alpha_0+1} [s(s-4m^2)]^{-1/2} \times \left[\left(\frac{s-2m^2}{4\mu^2-m^2}\right)^{\alpha_0+1} - 1\right] \theta(s-9\mu^2). \end{aligned}$$

This result imposes a bound on the leading Regge pole, according to

$$\alpha(0) \leq -\tfrac{3}{2} + [\tfrac{1}{4} + (g^2/16\pi^2 m^2)]^{1/2}. \quad (16)$$

For small values of g^2 this agrees with (6), which in turn agrees with the exact weak coupling limit. But for large values of g^2 (16) represents a considerable improvement over (6). The exact expression for $\alpha(0)$ in the ladder approximation, of course, is not known.

What is especially interesting is that (16) reveals for the *strong* coupling limit a bound on $\alpha(0)$ which grows with the *square root* of g^2. We now wish to argue that this behavior for large g^2 is in fact plausible for the true $\alpha(0)$ and that our bound is therefore a good one, apart from constant factors, in the limit of large g^2.

We consider the analogy to the Yukawa potential Here the effective potential, including the centrifugal barrier term, is

$$V_{\text{eff}}=-(\lambda/r)e^{-\mu r}+[\alpha(\alpha+1)/r^2].$$

For fixed binding energy the ratio $\alpha(\alpha+1)/\lambda$ clearly cannot increase indefinitely as $\lambda\rightarrow\infty$ because V_{eff} would then eventually become repulsive for all values of r and could not maintain a fixed bound state. Similarly $\alpha(\alpha+1)/\lambda$ cannot decrease indefinitely towards zero as $\lambda\rightarrow\infty$ because V_{eff} would then grow more and more attractive over an increasingly large range of r. In fact it is easy to conclude that, in order for a fixed bound state to be maintained, it is necessary that

$$\lim_{\lambda\rightarrow\infty}\frac{\alpha(\alpha+1)}{\lambda}=\frac{1}{\mu e},\quad e=2.7183\cdots.$$

This corresponds to the situation where the two zeros of V_{eff} approach each other as $\lambda\rightarrow\infty$, while the depth of the potential between them grows indefinitely. This property is quite general[14]: for any attractive potential that is less singular than r^{-2} at the origin and that falls off more rapidly than r^{-2} at infinity, $\alpha(\lambda)$ must satisfy

$$\lim_{\lambda\rightarrow\infty}\frac{\alpha(\alpha+1)}{\lambda}=\text{const}>0,$$

the constant depending on the shape of the potential.

[14] This has been noted independently by R. Blankenbecler (private communication).

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Approximation Techniques in Three-Body Scattering Theory*

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With the aid of some operator algebra the Lippmann-Schwinger integral equations for three-body transition amplitudes are recast in a form which involves two-body transition operators rather than two-body potentials. These equations, which are uncoupled and apply to all channels, are ideally suited to be the basis for approximation schemes, of the impulse approximation type, which have the distinctive feature of preserving unitarity. Two such approximations are described. With either of these as the leading term, a method of successive approximations is developed which yields an expansion for the exact amplitude whose convergence properties are expected to be considerably improved over the usual Born and multiple-scattering expansions. At high energies and low momentum transfers we obtain a unitary version of the strip approximation. Here the integral equation is quite tractable and represents the nondispersion-theoretic analog of multiparticle N/D techniques which have been applied recently to N-N and π-N reactions.

1. INTRODUCTION

In a previous paper[1] we have formulated a scheme for calculating three-body scattering amplitudes which generalizes the well-known impulse approximation by taking into account the constraints imposed by unitarity; effectively, one has summed an infinite set of diagrams of the impulse approximation type. A generalized N/D procedure was employed, in a model in which the incident particle interacts with only one of the target particles. An alternative to the N/D procedure which is in fact much more convenient and direct, particularly when none of the two-body potentials are ignored, will be described here. We again obtain amplitudes which satisfy a generalized unitarity relation which, however, can be derived without reliance on the multiple scattering expansions employed in Ref. 1. In fact, in Sec. 2, we derive the exact integral equations whose iterations give rise to the multiple scattering expansions. These integral equations are essentially the Lippmann-Schwinger equations recast, with the aid of some operator algebra, into a form which involves the two-body T operator, rather than the two-body potential. Such a reformulation is particularly desirable in the light of the observation[2] that the ordinary Born expansion of the three-body amplitudes in powers of the two-body potentials is essentially useless as a calculational tool. Similar T-operator integral equations were obtained earlier by Faddeev.[3] In the form given here they lend

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[3] L. D. Faddeev, Zh. Eksperim. i Teor. Fiz. **39**, 1459 (1960) [English transl.: Soviet Phys.—JETP **12**, 1014 (1961)]. These equations are highly coupled; they take the form of matrix inetgral equations. A more compact form, applicable to many-particle scattering problems, has been developed by S. Weinberg, Phys. Rev. **133**, B232 (1964), although the two-body potential still appears in Weinberg's formulation. Our equations, restricted here to the three-body case, combine the advantages of being uncoupled and potential-independent.